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ICE BREAKUP CHARACTERISTICS OF THE NASHWAAK RIVER AT DURHAM BRIDGE, N. B.

By

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ABSTRACT

At the request of the New Brunswick Subcommittee on River Ice, a study was initiated jointly by the National Water Research Institute and Water Survey of Canada (Fredericton), to explore whether existing hydrometric gauge records can be utilized to forecast the onset and severity of ice breakup. The gauge on the Nashwaak River at Durham Bridge, N. B., was selected for record examination and analysis. The results support the use of water stage as an index of breakup initiation and severity. It is concluded that useful, though incomplete, information can be extracted from existing records. The value of this information would be greatly enhanced by frequent qualitative descriptions of ice conditions during breakup and freeze up; by performing one or more discharge measurements during breakup; and by more complete measurements of ice thickness.

RÉSUMÉ

À la demande du sous-comité du Nouveau-Brunswick sur la glace des rivières, l'Institut national de recherche sur l'eau et les Relevés hydrologiques du Canada de Fredericton ont commencé ensemble une étude en vue d'établir s'il est possible d'utiliser les relevés par jauge hydrométrique existants pour prévoir le début et la gravité du dégel. La jauge de la rivière Nashwaak installée à Durham Bridge au N.-B. a été choisie en vue d'étudier et d'analyser les relevés. Les résultats appuient l'usage du niveau de l'eau comme indice du début et de la gravité du dégel. On a conclu qu'on peut extraire des renseignements utiles, quoique incomplets, des relevés existants. La valeur de ces renseignements serait grandement renforcée par des descriptions qualitatives fréquentes des conditions de la glace durant le dégel et le gel, par la prise d'une ou de plusieurs mesures des débits d'écoulement au cours du dégel et par des mesures plus complètes de l'épaisseur de la glace.

MANAGEMENT PERSPECTIVE

The timing of river ice breakup and the maximum water stage at the time of breakup are very useful information, especially with respect to flood protection. This report shows that existing hydrometric gauge records can be used to forecast the onset and severity of ice breakup at that particular site.

In order to develop general forecasting methods, it will be necessary to have case studies of many more sites. In this respect, the data gathered by Water Survey of Canada could be extended by the addition of qualitative descriptions of ice conditions; by making more discharge measurements during breakup and by making more complete measurments of ice thickness.

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September 7, 1982

PERSPECTIVE DE GESTION

Le moment du dégel des rivières et le niveau d'eau maximal à ce moment constituent des renseignements très utiles, surtout pour se protéger contre les crues des rivières. D'après le présent rapport, on peut se servir des relevés par jauge hydrométrique existants pour prévoir le début et la gravité du dégel à un endroit précis.

Afin d'élaborer des méthodes générales de prévision, il faudra étudier un bien plus grand nombre d'emplacements. À cet égard, on peut améliorer les données réunies par les Relevés hydrologiques du Canada en y ajoutant des descriptions qualitatives des conditions de la glace, en mesurant davantage les débits d'écoulement pendant le dégel et en mesurant de façon plus complète l'épaisseur de la glace.

T. Milne Dick
Chef, Division de l'hydraulique
le 7 septembre 1982

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1.0 INTRODUCTION

During the summer 1980 meeting of the N.B. Subcommittee on River Ice (formerly: Ad Hoc Committee on Ice and Ice Jams), a question arose as to whether existing hydrometric gauge records can be utilized to forecast the initiation and severity of river ice breakup. To explore this possibility, a joint (NWRI/WSC) study was initiated for the hydrometric gauge located on the Nashwaak River at Durham Bridge in New Brunswick. The chief objective of this study is to determine whether the gauge record can provide information that may be used to develop a method to forecast the initiation and severity of breakup. The results of the study to date are reported herein.

2.0 DESCRIPTION OF DATA

The main source of data has been the WSC record of gauge height versus time for the period 1965-81. Supplementary information consisted of daily discharge data (WSC), meteorological data (Atmospheric Environment - Monthly Records") and channel hydraulics in the vicinity of the gauge (B. Burrell, N. B Environment). From these data, several parameters thought to be characteristic of the ice regime have been extracted as described below.

2.1 Maximum Stable Freeze Up Stage (H_F)

A typical but not universal configuration of the daily average stage hydrograph near the start of the ice season is sketched in Fig. 1. The solid line represents the actual stage whereas the broken line gives the "effective" stage (=stage that would have occurred had the flow been unaffected by ice). At a certain time which may be termed the beginning of freeze up, the actual stage begins to rise while the effective stage continues to drop. Eventually, the actual stage attains a peak and then declines. This sequence reflects the dynamic nature of ice cover formation in a river. With the onset of cold weather, frazil ice forms and is, initially, transported freely. The effect of this frazil ice on the water stage is negligible. As more and more frazil is produced, it begins to agglomerate into slush and pancakes. Eventually, the ice transport is impeded somewhere downstream of the gauge and an ice cover begins to propagate upstream. The presence of the ice cover causes a local stage increase which eventually begins to be "felt" at the gauge site. The gauge height then increases until the time when the edge of the ice cover arrives at the gauge site. Subsequently, the gauge height decreases due to the combined effects of decreasing discharge and thermal smoothing of the underside of the cover. The peak stage (H_F) during this period is considered an important factor influencing the succeeding breakup because it defines the stage at which the ice cover is formed; the width of the cover is approximately equal to the channel width at the stage H_F. To eliminate brief peaks during which there is little time for freezing, H_F is defined as a daily average value. It should be recognized that the above description may not apply in cases where there are severe flow and stage controls; where conditions are such that a complete ice cover does not form; or where the discharge is decreasing so rapidly as to suppress the occurrence of a peak on the daily stage hydrograph.

2.2 "Winter" Peaks

Occasionally, a brief thaw may occur during the winter period. If such a thaw causes significant runoff, the gauge record will show a peak which may or may not initiate breakup. In the latter case, the peak stage represents a lower limit for the stage required to initiate breakup at that time.

2.3 Stage at Initiation of Breakup (H_B)

When a thaw does lead to breakup of the ice cover, the gauge record shows irregularities that cannot be explained by discharge variations, as illustrated in Fig. 2. Typically, the stage begins to rise from its fairly steady winter value and shortly afterwards exhibits spikes and peaks, even though the discharge (and thence the effective stage) varies smoothly. These irregularities are due to breaking or broken ice effects. A probable value for the stage at the initiation of breakup may be fixed at the first significant spike[1]. Unfortunately, this definition is not always objective because there may be instances where it is difficult to decide whether a spike on the gauge record is significant. Only a probable range of H_B can then be determined: the lower limit of this range is the stage before the rise starts and the upper limit is the first peak that can be attributed to broken ice effects. Clearly, the uncertainty associated with the determination of H_B would be greatly reduced by simultaneous visual observations of ice conditions but such information is mostly absent in the present study.

2.4 Maximum Breakup Stage (H_m)

This is the maximum stage reached during the breakup period and its determination is straight-forward (see Fig. 2).

2.5 Maximum Ice Effect on Stage (ΔH_m)

The ice effect on stage is the difference between the actual stage and the effective stage. The time of maximum ice effect can usually be determined by simple inspection (Fig. 2) and does not necessarily coincide with the time of H_m.

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[1] Initiation of breakup is defined herein as the instant when a sustained movement of the ice cover begins. When a stationary ice cover is set in motion, the stage drops as a result of reduced resistance to flow.

2.6 Discharge and Effective Stage

Daily average discharge values are estimated by WSC based on interpolations between discharge measurements as well as on such evidence as upstream and tributary flows, runoff and weather conditions, etc. Such estimates may involve considerable error. This has repercussions on the accuracy of the effective stage which is determined by joining daily values plotted at noon of each day. For the Nashwaak River at Durham bridge, the effective stage error is probably less than ±0.3 m during periods of steady conditions but could exceed this value otherwise. (D. Randall, personal communication).

2.7 Ice Thickness (h_i)

Ice thickness has been estimated from WSC discharge measurement notes (Fredericton office). Such notes give the distance from the water surface to the bottom of the ice which, under free flotation conditions, represents 92% of the total ice thickness. However, this assumption may or may not be valid depending on whether there is significant bank support of the ice or deep snow cover which may cause the free water surface to rise above the top of the ice. The presence of a slush deposit under the solid ice may render thickness values completely unreliable because the notes would then show the distance from the water surface to the bottom of the slush. Another source of error may be (unreported) instances when "water surface" has been used nominally, i.e. substituted by a more convenient datum such as the top of a deep snow layer. (D. Randall, personal communication).

2.8 Total Heat Input to the Outer Ice Surface (Σq).

This is the sum of daily values (q), starting on the first day for which q is positive and ending at the time of breakup initiation. Summation is carried out algebraically during this period, so long as Σq remains positive. If Σq becomes negative, it is set equal to zero and summation starts again on the next day of positive q. It is emphasized that Σq is an index rather than an actual measure of the total amount of heat absorbed by the ice cover prior to breakup (see Appendix A for details of definition and computation of q and Σq). In short, q is calculated from the following equation:

$$q = a\Theta + bS_u + c \tag{1}$$

in which q = the daily amount of heat in J/cm^2; θ = daily average air temperature in °C; S_u = total hours of bright sunshine during the day; and a, b, c are coefficients which depend on latitude and time of year, as tabulated below (latitude ≈ 46°N).

TABLE 1. Coefficients to be Used in Equation 1
(see Appendix A for explanation)

Time of Year Coefficient	Jan. 15	Feb. 15	Mar. 15	Apr. 15
a	52.8	61.2	68.7	78.8
b	5.9	15.5	26.8	35.2
c	12.6	74.6	160.1	250.6

Introduction of Σq as a pertinent parameter was thought desirable after performing a preliminary analysis which showed significant influences by both the accumulated degree-days of thaw and the accumulated hours of sunshine. Use of Σq has the advantages of reducing the number of the thermal parameters from two to one and accounting for the effects of the time of year. Usually, Σq is calculated up to and including the day on which breakup is initiated. However, in cases where this event occurred early in the day, the summation was terminated on the previous day.

3.0 ANALYSIS OF DATA

Table 2 summarizes the present data for the Nashwaak River at Durham Bridge. Of the 21 freeze up - breakup events that occurred during the period of record (1965-81), one has proved impossible to interpret (1964-65) while seven presented serious difficulties (see last column of Table 2).

3.1 Initiation of Breakup

It has been suggested (Shulyakovsky 1963) that, at a given site, the rise (H_B - H_F) required to initiate breakup of the ice cover, depends on ice thickness (h_i) and total amount of heat input to the ice cover (Σq). Table 2 indicates that only a few h_i's are available which, at the same time, show no large variability. It was therefore decided to ignore the effects of h_i at this time. In the future, it may be possible to generate applicable h_i's based on correlations that may be developed between h_i and time since freeze up or corresponding accumulated degree-days of frost. Figure 3 shows (H_B-H_F) plotted versus Σq. Probable values of (H_B-H_F) are represented by solid circles; ranges of (H_B-H_F) are designated by vertical or inclined straight lines depending on whether the corresponding limits of H_B occurred on the same or different days; arrows indicate known limits of H_B; uncertain data points are

TABLE 2. SUMMARY OF BREAKUP CHARACTERISTICS AND ASSOCIATED PARAMETERS; NASHWAAK R. AT DURHAM BRIDGE 1965-81

Season	H_F	Time of H_F	Winter Peaks			Breakup					Ice Thickness		Maximum Stage			Maximum Ice Effect				Remarks
	(m)	(day/month)	H (m)	Time of H	Σq (J/cm^2)	Range of H_B (m)	Probable H_B (feet)	Time of H_B	Σq (J/cm^2)	Σq Calculated to (day/mo.)	h_i (m)	Time of Measurement	H_m (m)	Time of H_m	Discharge (m^3/s)	ΔH_m (m)	Time of ΔH_m	H (m)	Discharge (m^3/s)	
																				Breakup undefinable
964-65	2.56	13/1	2.20	7/3	2263		1.58	22/3	2807	22/3			2.23	26/3	83	0.55	22/3	1.59	22	
965-66	1.40	29/12					1.71	7/4	5992	7/4	0.70	16/3	1.73	11/4	15	0.82	11/4	1.73	15	
966-67	1.80	27/12																		
							3.25	4/2	461	4/2	0.37	12/1	3.87	4/2	100	2.05	4/2	3.87	100	h_i not representative of breakup value
967-68	2.34	1/1																		
1967-68	3.35	6/2	2.78	25/3	3855		2.71	30/3	6411	30/3			3.40	1/4	93	1.65	1/4	3.40	93	Difficult to define breakup
1968-69	2.25	29/12	1.84	27/1	0		1.87	28/3	3562	28/3			1.87	30/3	22	0.82	30/3	1.87	22	
1969-70	3.44	1/1				1.98 - 5.31		2-4/2	587	3/2			5.31	4/2	697	0.92	4/2	5.31	697	- " -
1969-70	2.29	16/2				1.43 - 1.80		27/3	4819	27/3			2.05	28/3	34	0.82	28/3	2.05	34	- " -
1970-71	1.65	10/12	2.07	15/2	251	0.90 - 1.31		3-4/4	6872	3/4	0.64	11/3	1.96	14/4	49	0.93	6/4	1.79	30	
			1.83	18/3	2179				7542	4/4										
1971-72	1.19	7/12	1.34	13/12	251	1.62 - 2.19		17-19/3	1048	17/3	0.73	6/3	2.49	20/3	28	1.39	20/3	2.49	28	
			1.43	16/1	0				1467	18/3										
			1.59	16/2	1383															
			1.74	12/3	0															
1972-73	2.72	14/12	2.79	5/2	0	1.91 - 2.35		17-18/3	5363	18/3			2.61	19/3	76	1.27	18/3	2.53	40	
			2.16	13/3	3352															
1973-74	2.20	10/1	2.22	8/3	3017	1.56 - 2.19		4-7/4	2682	4/4	0.70	27/3	2.19	7/4	56	0.76	7/4	2.19	56	
									4735	7/4										
1974-75	2.19	4/1	2.15	22/3	2137	1.48 - 1.68		4-8/4	4483	4/4			1.68	8/4	32	0.52	8/4	1.68	32	Difficult to define breakup
									6034	8/4										
1975-76	2.44	14/12	3.38	29/1	277		2.19	27/3	4064	26627/3(avg)	0.73	2/3	2.94	29/3	126	1.15	30/3	2.78	126	
1976-77	1.89	12/12	2.69	27/12	0	1.49 - 1.53		27-28/3	6411	27/3			2.29	31/3	74	0.69	31/3	2.29	74	Difficult to define H_F and H_B; H_F= 2.29 on 29/12 might be better
			1.91	16/3	3645															
1977-78	1.82	11/12				2.15 - 2.51		26/12	21	26/12			2.51	26/12	42	1.23	26/12	2.51	42	Probably thin ice
1977-78	2.74	18/1				1.42 - 1.54		11/4	7123	11/4			1.76	15/4	47	0.44	15/4	1.76	47	Difficult to define H_F
1978-79	1.10	27/11				< 6.33		3/1	293	2/1			2.57	3/1	74	0.96	3/1	2.57	74	
1978-79	3.40	7/1	3.26	31/1	1089	1.54 - 2.85		5-8/3	922	5/3			3.20	26/3	207	1.15	15/3	2.76	74	Difficult to define breakup
									2179	7/3										
1979-80	2.25	4/1				2.71 - 3.01		18-19/3	629	18/3	0.55	13/3	3.03	19/3	105	1.65	1/4	2.84	36	
1980-81	2.41	17/12				1.81 - 2.11		19-23/2	712	18/2			2.12	23/2	68	0.57	23/2	2.12	68	
									2556	22/2										

identified by inclined strokes. Despite considerable scatter[2], Fig. 3 shows that $(H_B - H_F)$ decreases with increasing Σq as might have been expected. Large deviations from the average line drawn in Fig. 3 are, with one exception, associated with uncertain data. For "premature" breakup events ($\Sigma q = 0$) a rise of about 0.9 m over H_F is necessary to initiate breakup. On the other hand, "mature" events ($\Sigma q > 6300$ J/cm^2) are initiated at a stage about 0.6 m below H_F.

3.2 Maximum Stage During Breakup

Maximum breakup stages (H_m) are plotted versus H_F in Fig. 4 where a trend for H_m to increase with H_F is evident. The upper envelope drawn in Fig. 4 has the equation (in metric units)

$$H_m = 1.22 + 1.18\ H_F \text{ (upper envelope)} \qquad (2)$$

The actual value of H_m is generally less than that given by Equation 2, depending on several factors in addition to H_F. For example, Σq, h_i and flow discharge are obviously important in this regard. Moreover, H_m also depends on whether and where (relative to the gauge site) ice jams form. Considering the thermal effect, it is reasonable to expect that increasing values of Σq cause increasing deviation of H_m from the upper envelope of Fig. 4. This is confirmed in Fig. 5 where the difference $[H_m - (1.18\ H_F + 1.22)]$ has been plotted versus Σq. The upper envelope drawn in Fig. 5 provides a means of improving short-term forecasts of flooding potential in cases where, in addition to H_F, Σq can be estimated or is known.

The effect of discharge on stage is illustrated in Fig. 6 where the stage at H_m and ΔH_m is plotted versus the prevailing

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[2] Presumably, a part of the scatter is due to, as yet unknown, effects of ice thickness

discharge. [Note that stage in Fig. 6 is taken equal to gauge height minus 0.30 m, the latter being the approximate value of gauge height at which discharge is zero[3].] In view of earlier comments on the accuracy of discharge and effective stage (Section 2.6), Fig. 6 can only be regarded as a qualitative indication of trends. Also shown in Fig. 6 is the predicted stage-discharge relationship based on the theory of floating equilibrium jams of the "wide" channel kind (Pariset et al 1966; Beltaos 1982). Average channel depth was determined as a function of gauge height using three cross-sections located downstream of the gauge site (provided by B. Burrell). River slope and width were taken as 0.73 m/km and 67 m respectively. Because a constant width has been assumed, the effect of overbank flow has not been taken into account. This effect should increase with increasing stage and its consideration would require detailed cross-sectional data and a more elaborate analysis.

The values of H_m shown in Table 2 may also be used to perform a frequency analysis which can be helpful in long-term forecasting. Figure 7 is a dimensionless stage-probability graph. The net gauge height H'_m (=H_m-0.3 m) is divided by $(H'_m)_{50}$ (=value of H'_m equalled or exceeded 50% of the time) and plotted versus the corresponding probability, along with similar data for three other river sites. Because the mathematical form of the probability distribution of H'_m is unknown, it was thought preferable, for the present, to use a simple arithmetic graph in Fig. 7 rather than a special probability graph such as normal or log-normal. It is noteworthy that the data points in Fig. 7 tend to define a single curve, though no theoretical reason is known to the writers why this should be the case.

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[3] This value, say x, was determined by trial and error as that which gives the best fit when discharge is assumed to vary in proportion to a power of gauge height minus x. The value of gauge height reduced by x provides a rough measure of the flow depth.

In closing this section, it is emphasized that the graphs and relationships presented herein are, to a large degree, empirical. Therefore, the present results are site-specific; quantitative extrapolation to other sites is not justifiable. Clearly, the task of developing general forecasting methods which could be applied to sites for which no records are available, would be greatly facilitated by accumulating and comparing several case studies such as the present.

4.0 EXAMPLE OF APPLICATION

It is known that in a given season, the maximum freeze up stage (H_F) is 2.44 m. On March 14, a warm weather trend and increased runoff are forecast for the next few days, as indicated below.

Date in March	Air Temperature (°C)	Hours of Sunshine	Gauge Height (m)
Known:			
13	-5	0	1.52
14	-2	4	1.52
Forecast:			
15	2	5	1.83
16	4	8	2.44
17	6	10	3.35

To calculate Σq, we use Eq. 1. From Table 1, we obtain for March 15: a = 68.7, b = 26.8, c = 160.1. With these values, we compute q = negative on March 13; and q = 130, 432, 649 and 840 for the subsequent four days (March 14 to 17). Corresponding values of Σq and H_B (using Fig. 3) are as follows.

Date in March	Σq (J/cm^2)	H_B (m)	Gauge Height (m)
13	0	3.35	1.52
14	130	3.32	1.52
15	562	3.20	1.83
16	1211	3.02	2.44
17	2051	2.74	3.35

Breakup is thus expected to start sometime during March 17, this being the first date on which H_B is less than the expected gauge height. Assuming that the applicable value of Σq is the average of the March 16 and 17 values (note that the average H_B is approximately equal to the corresponding average gauge height), gives $\Sigma q = 1631$ J/cm^2. Figure 5 then gives $H_m - 1.22 - 1.18\ H_F \leq -0.15$ m). It follows that H_m should not exceed 3.9 m.

5.0 SUMMARY AND CONCLUSIONS

The present analysis supports the use of water stage as an index to forecast breakup initiation and severity. At the same time, it has been found that useful, though incomplete, information can be extracted from existing hydrometric gauge records. The value of this information would be greatly enhanced by frequent qualitative descriptions of ice conditions during breakup and (less importantly) freeze up; by performing one or more discharge measurements during breakup; and by more complete measurements of ice thickness. At present, forecasting methods developed on the basis of past records are site-specific. Additional studies at other sites would contribute to the development of more general methodology which could be applied to sites without records.

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APPENDIX A.

Calculation of Σq

The daily amount of heat input per unit outer area of the ice cover may be calculated according to the method described by Shulyakovsky 1963 (note that heat input by the water per unit inner area of the cover is ignored).

$$q = Q + I_{eff} + P + LE \tag{A.1}$$

in which q = total heat exchange in J/cm^2 day; Q = short wave radiation; I_{eff} = long-wave radiation; and $LE + P$ = heat exchange by conduction and convection plus evaporation or condensation. Shulyakovsky indicated that experience in breakup forecasting has shown that it is best to only allow for the heat exchange during the daytime and ignore that of the night time. The heat exchange components are then computed as follows:

$$Q = 0.5\, Q'\, \alpha \tag{A.2}$$

in which Q' = maximum possible solar radiation (can be determined from Shulyakovsky's Table 30 as a function of latitude and time of year); and α = dimensionless coefficient which, according to Shulyakovsky, depends on low-level (N_l) and total (N_t) cloudiness, i.e.

$$\alpha = 1 - 0.47\, N_l - 0.20\, N_t \tag{A.3}$$

A more convenient expression that only involves the total cloudiness is given by List (1949):

$$\alpha = 0.96 - 0.61\, N_t \tag{A.4}$$

In Eq. A.2, the albedo of the ice surface has been fixed at 0.5 as a first approximation (Shulyakovsky 1963). It should be understood, however, that this is an average value. In reality, the albedo should be high at the beginning of the period under consideration and decline with the passage of time. The second component of q in Eq. A.1 is given by

$$I_{eff} = K_d [-2770 + \sigma T_a^4 \beta] \qquad (A.5)$$

in which k_d = length of daytime expressed as a fraction of the 24-hour period (can be determined from Shulyakovsky's Table 31 as a function of latitude and time of year); σT_a^4 is the black body radiation (σ = Stefan Boltzmann constant, T_a = air temperature in °K), given as a function of θ in Shulyakovsky's Table 6, or, for the normally encountered range of θ:

$$\sigma T_a^4 \simeq 2770 + 41.9 \theta \qquad (A.6)$$

in which θ = air temperature in °C. In Eq. A.5, the coefficient β is given by Shulyakovsky as

$$\beta = 0.76 + 0.11 N_l + 0.10 N_t \qquad (A.7)$$

A more convenient expression that only involves N_t was derived by simplifying an expression quoted by Tsang (1982), i.e.:

$$\beta \simeq 0.76 + 0.18 N_t \qquad (A.8)$$

Equation A.8 is based on assumed average values of 0.4kPa and 1.6 km for the vapour pressure and cloud height respectively. Substituting Eqs. A.6 and A.8 in Eq. A.5 gives

$$I_{eff} \simeq k_d \, [-665 + (31.8 + 7.5 \, N_t) \, \theta + 499 \, N_t] \tag{A.9}$$

Noting that N_t is between 0 and 1 so that the multiplier of θ is between 31.8 and 39.4, Eq. A.9 is simplified to:

$$I_{eff} \simeq K_d \, [-665 + 35.6 \, \theta + 499 \, N_t] \tag{A.10}$$

The third component of q in Eq. A.1 is given by

$$LE + P = K_d \, (29.4 + 16.1 \, w) \, (\theta + 15.4 \, e - 9.4) \tag{A.11}$$

in which w = (mean daytime) wind speed in m/s; and e = (mean daytime) vapour pressure in kPa. For the present study, weather data reported by Atmospheric Environment for Fredericton CDA have been used. At this station, an approximate relationship between e and θ was established for the normally encountered range of θ, i.e.:

$$e = 0.47 + 0.023 \, \theta \tag{A.12}$$

The main reason for introducing Eq. A.12 is convenience because the published data ("Monthy Records") do not give daily values of e. For the same reason, an assumed value of 3 m/s has been substituted for w. With these assumptions, Eq. A.11 reduces to

$$LE + P \simeq 77.7 \, K_d \, (1.35 \, \theta - 2.2) \tag{A.13}$$

Substituting the foregoing expressions in Eq. A.1 and rearranging gives:

$$q = a\theta + b \, S_u + c \tag{A.14}$$

in which S_u=number of hours of sunshine [note $N_t=1-(S_u/24K_d)$]; and a, b, c are coefficients defined by

$$a = 140\ K_d \tag{A.15}$$

$$b = (0.305\ Q' - 499\ K_d)/24\ K_d \tag{A.16}$$

$$c = 0.175\ Q' - 337\ K_d \tag{A.17}$$

The coefficients a, b and c depend on latitude and time of year. For the Nashwaak River at Durham Bridge, the latitude is approximately 46°N (Water Survey of Canada 1979) whereby Shulyakovsky's Tables give the values shown in Table 1. Though a, b and c change with date, it is a fair approximation to assume constant values over summation periods lasting no more than five days. For the present calculation, the mean daily air temperature (rather than the mean daytime value) has been used for convenience. This and other simplifications described earlier were thought justifiable for the present purpose, considering that Σq is no more than an index while the present study is exploratory. Refinements could be effected by:

(i) Using the mean daytime air temperature θ_d rather than the mean daily temperature.

(ii) Using the measured daytime wind speed w_d in place of the assumed value of 3 m/s. This would require access to unpublished weather data.

(iii) Using the measured daytime vapour pressure rather than the approximate relationship of Eq. A.12. This would also require access to unpublished data.

(iv) Using Q' values that are more applicable to the study site rather than Shulyakovsky's values.

(v) Using daily values of a, b and c rather than averages for the breakup period.

FIGURES

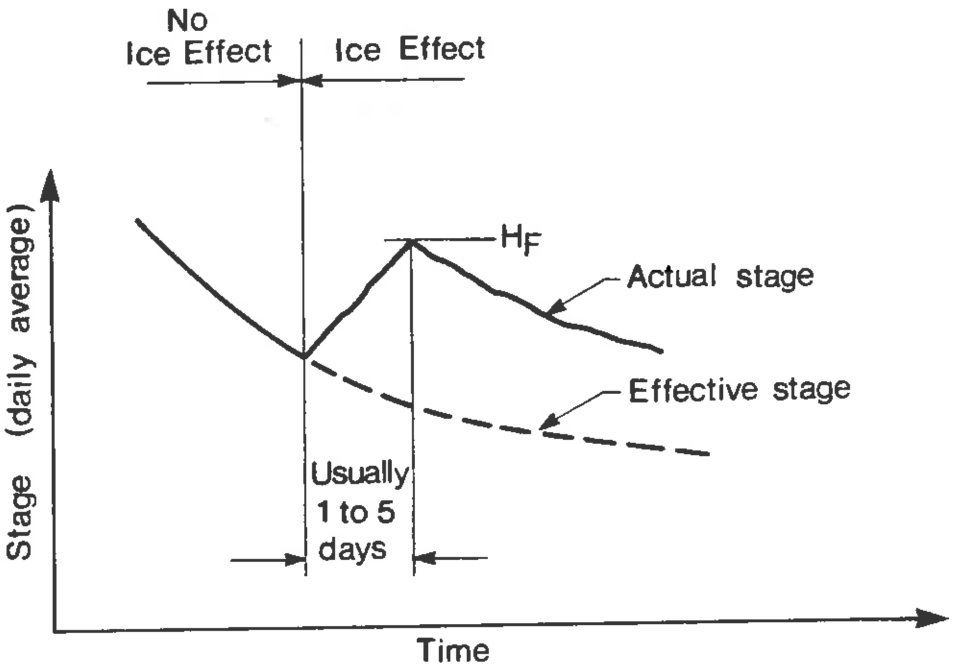

Fig. 1 Schematic illustration of daily stage variation with time during beginning of freeze up.

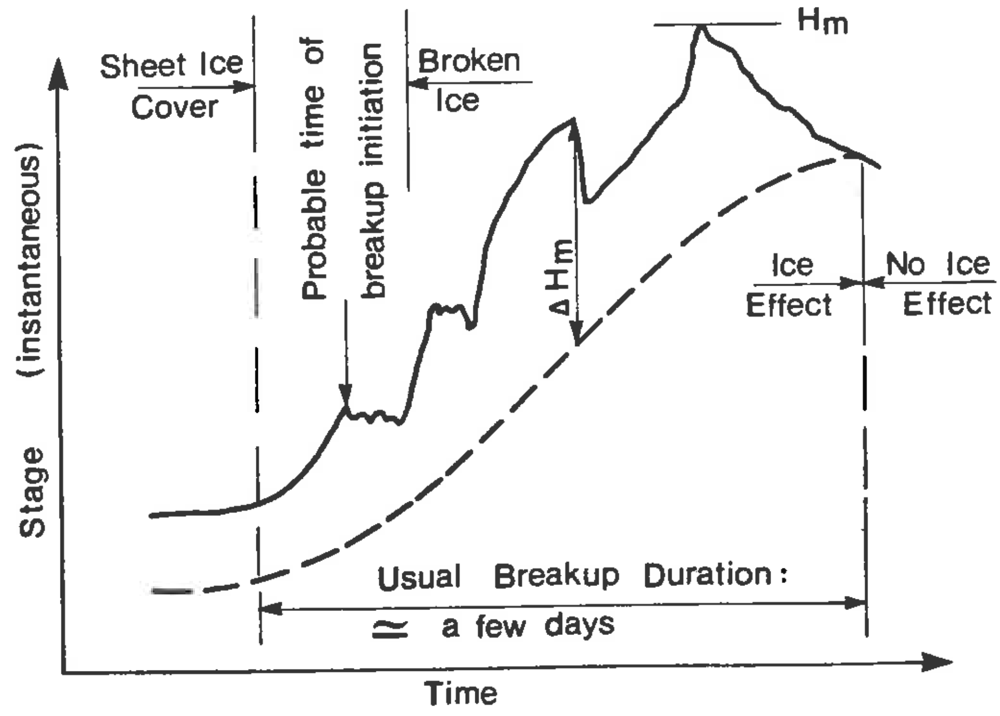

Fig. 2 Schematic illustration of instantaneous stage variation with time during breakup.

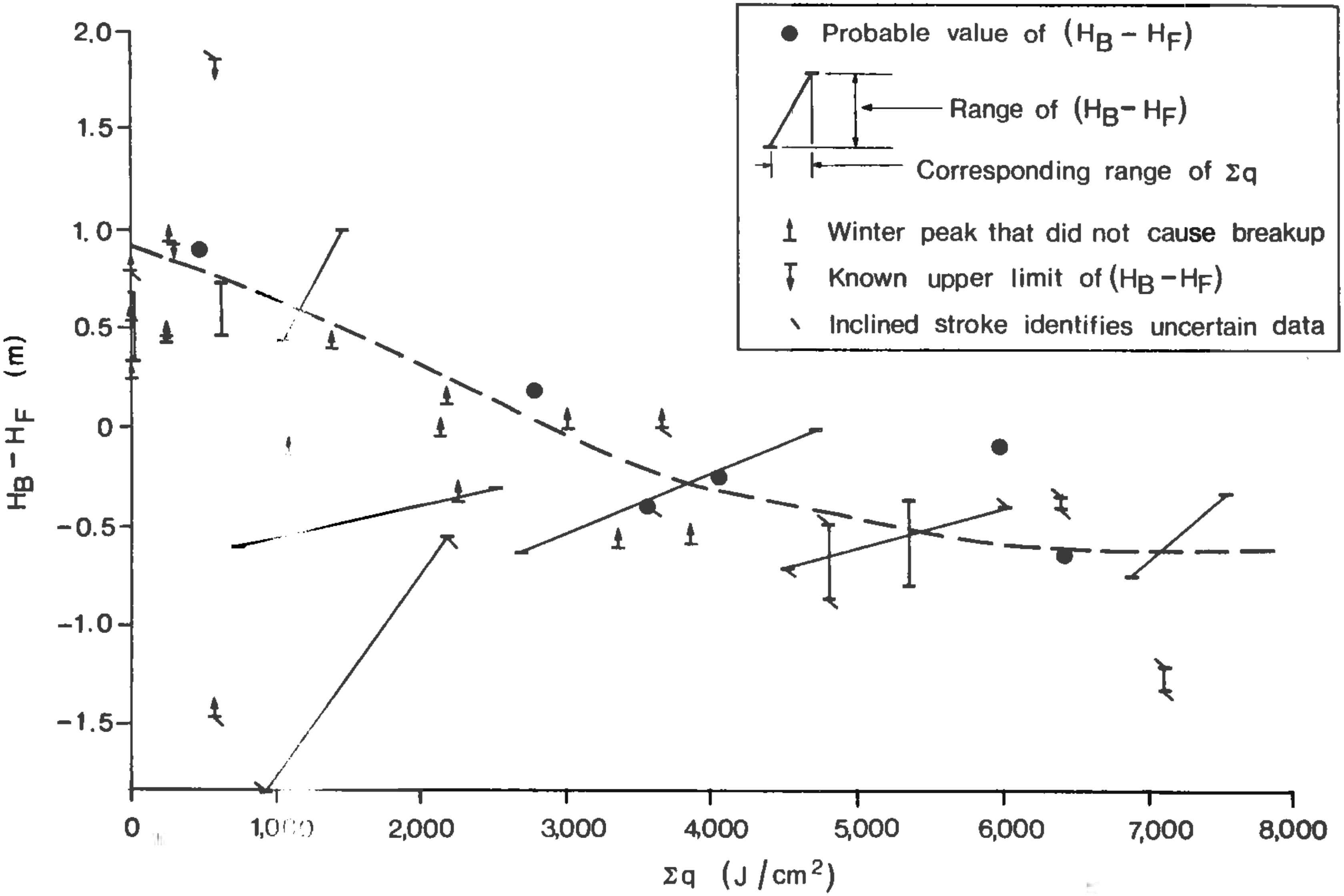

Fig. 3 Stage rise above H_F, required to initiate breakup, as a function of Σq.

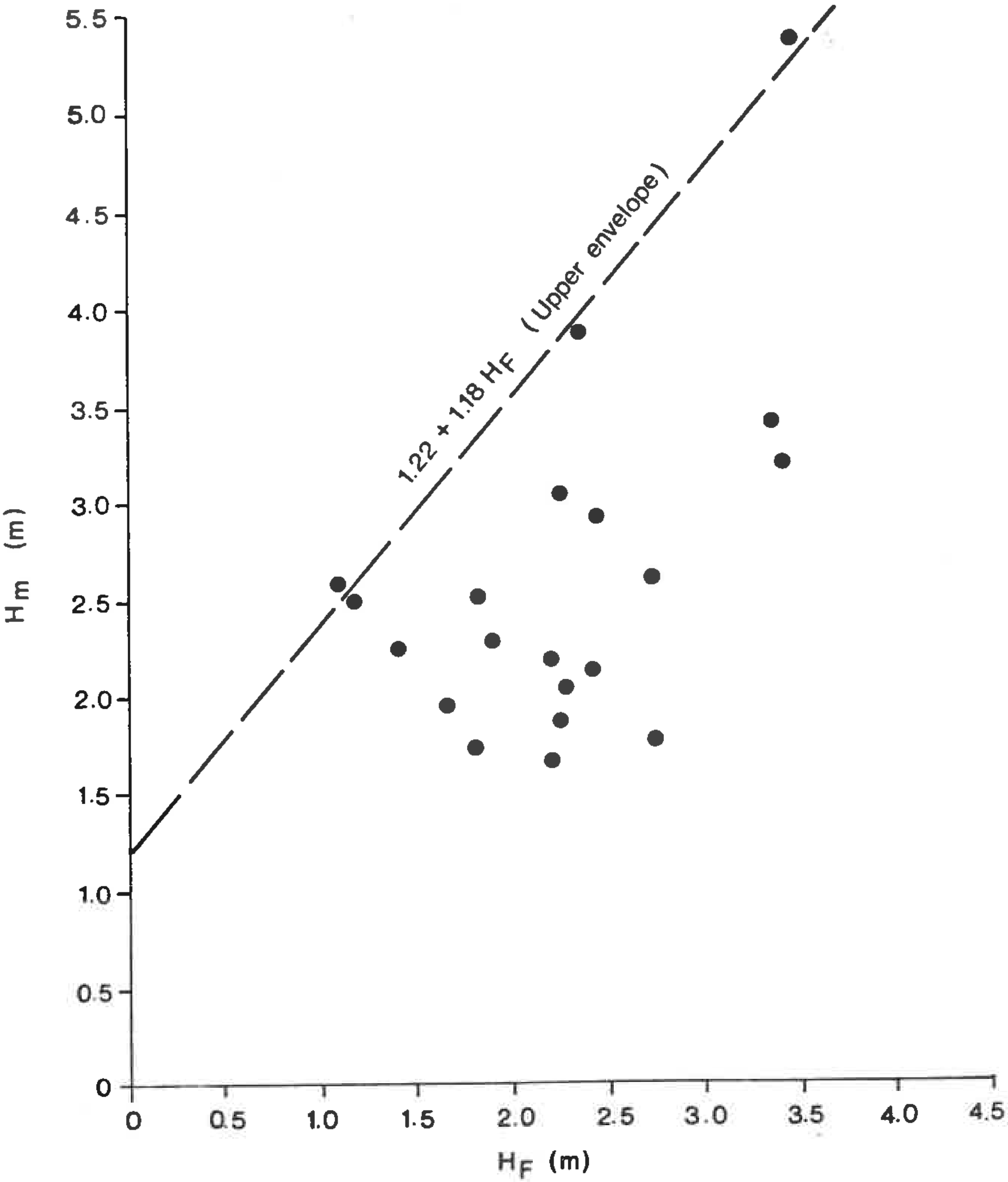

Fig. 4 Maximum stage during breakup versus H_F

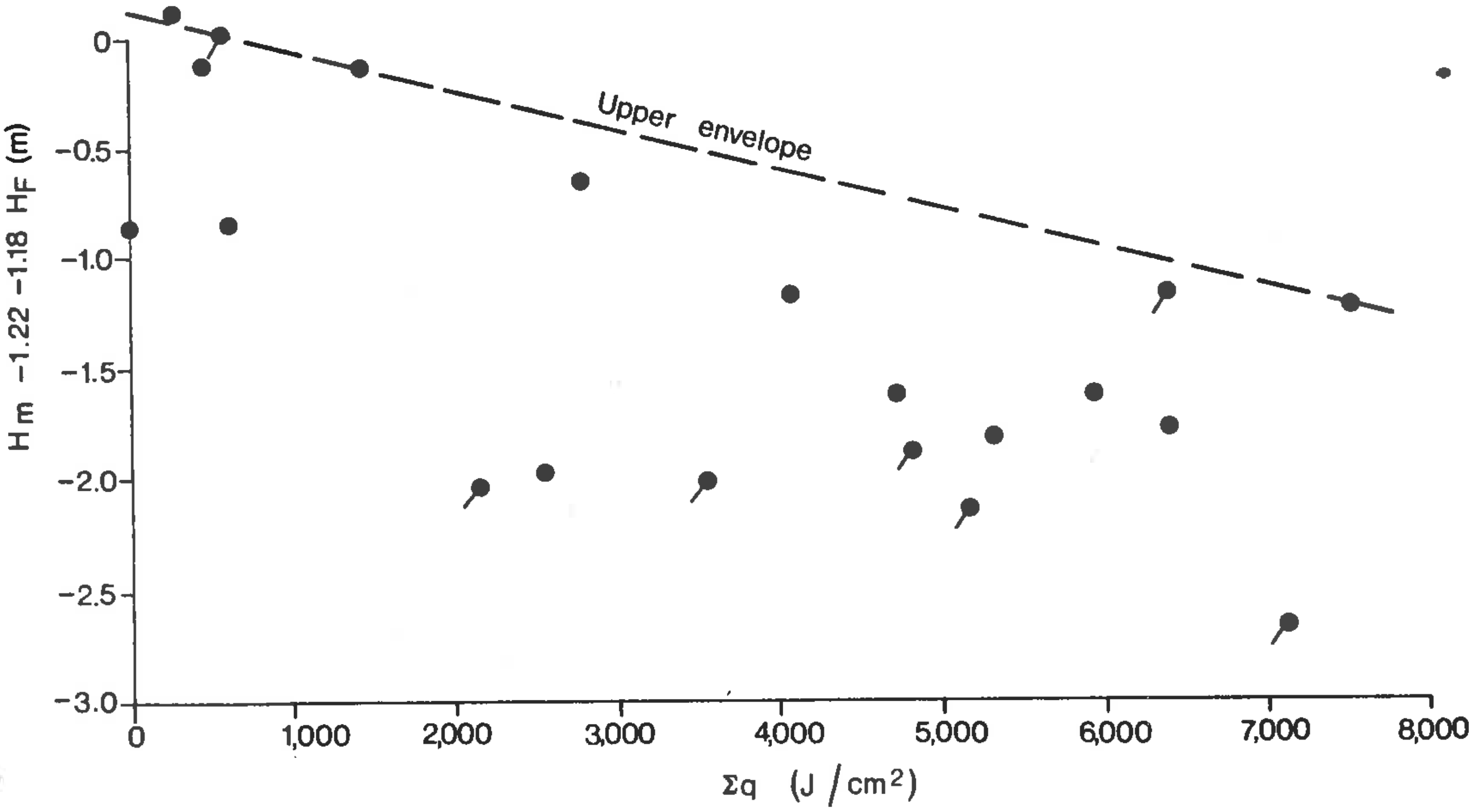

Fig. 5 Effect of Σq on H_m

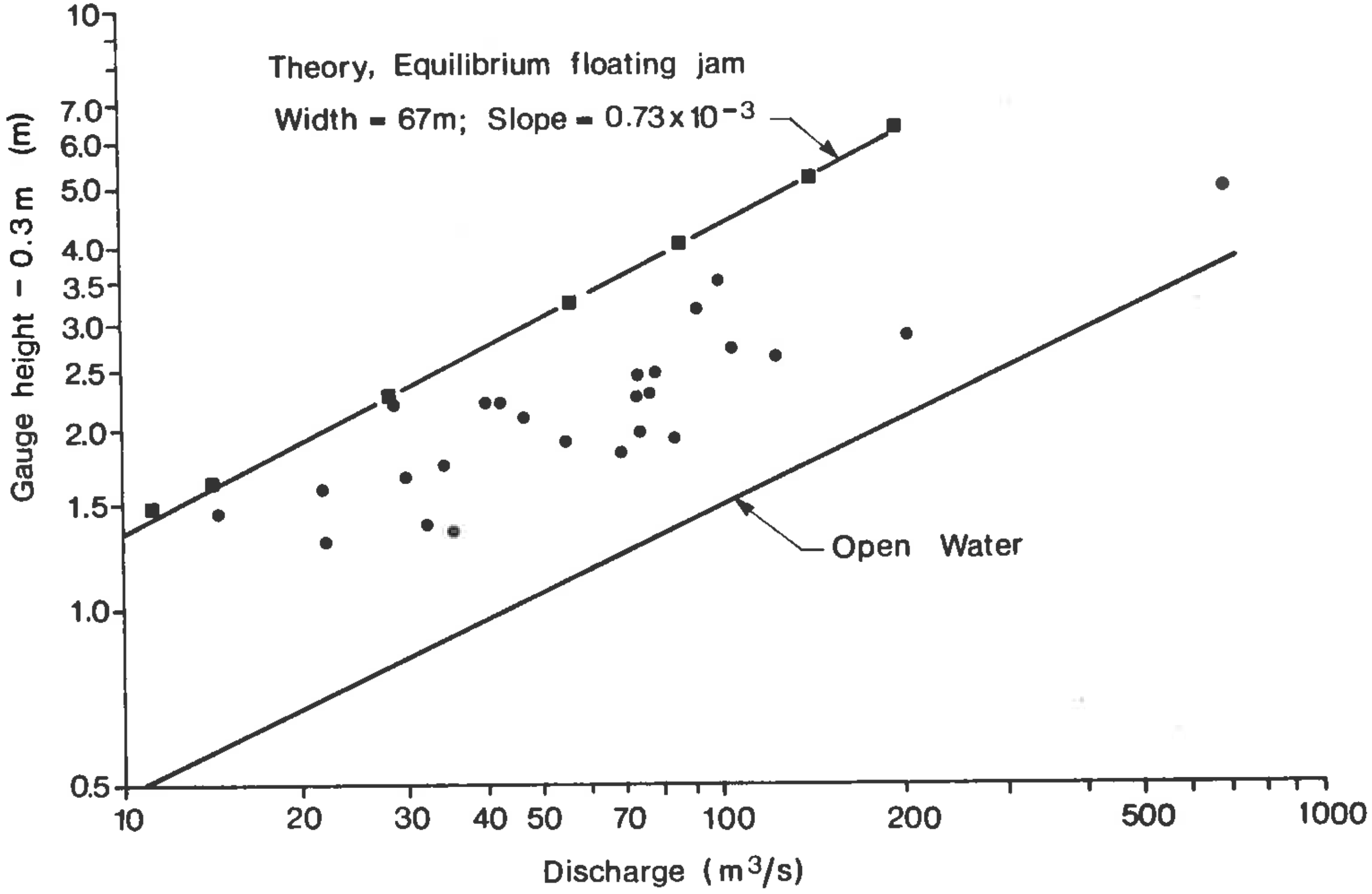

Fig. 6 Effect of discharge on stage during breakup

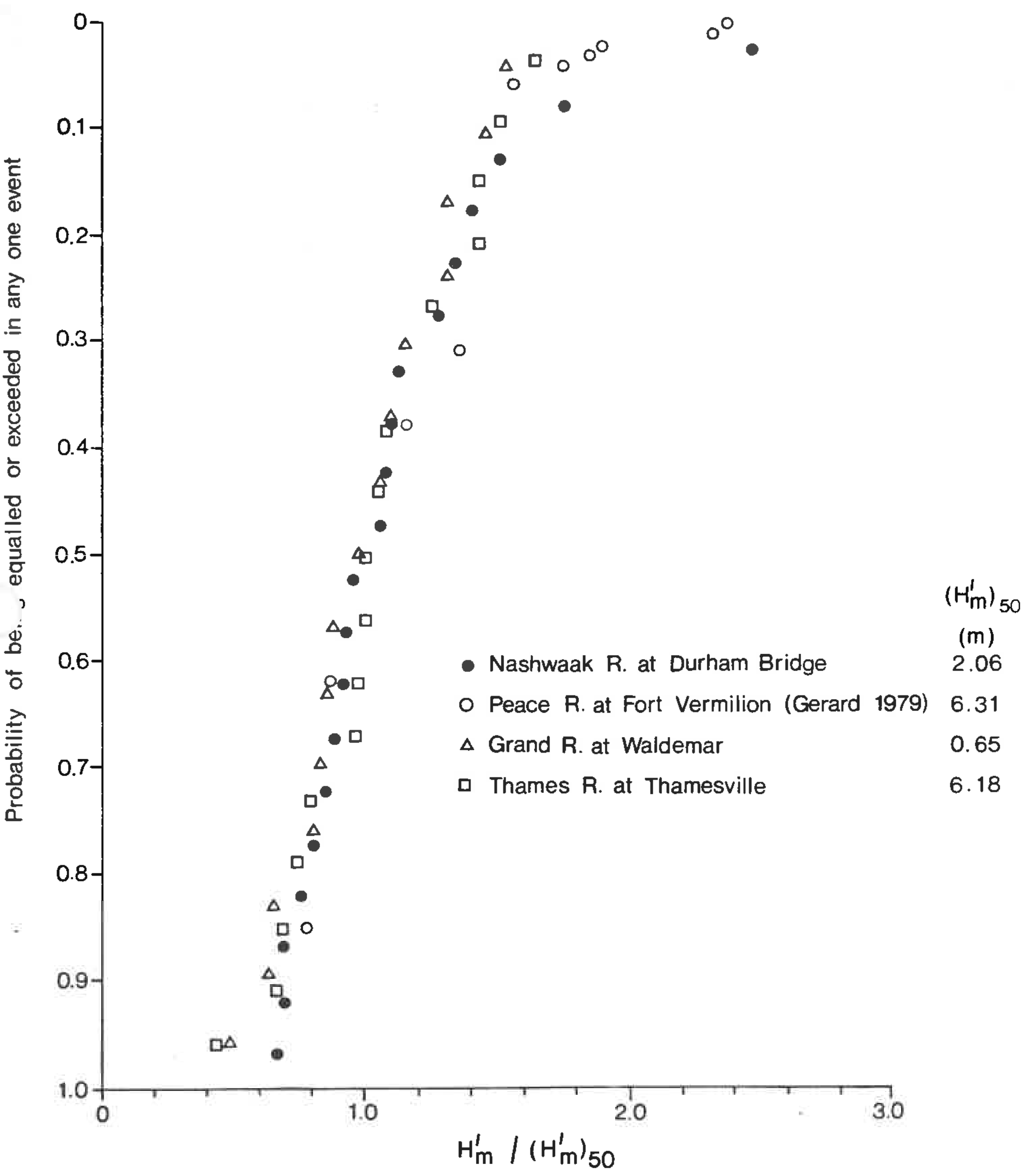

Fig. 7 Results of probability analysis